The network structure of the tuna commodity chain

Christian Mullon, Jeanne Fortilus, Michael Mansoor, Patrice Guillotreau

May 9, 2011
The tuna commodity chain
The tuna commodity chain

Entities

- $a$: areas
- $p$: countries
- $e$: species
- $f$: fresh or frozen commodity
- $c$: prepared commodity
Areas and species

The tuna commodity chain
Entities
Areas and species
Fleets and species
Countries and commodities
Picture
Picture
Balancing data
Pictures of the network
Modeling
Networks economics I
Network economics II
Balancing data

Available data

Network flows: Notation

Constraints

Constraints

Calibration

Constraints

Calibration

Balancing

Pictures of the network

Modeling

Networks economics I

Network economics II
Available data

The tuna commodity chain

Balancing data

Available data

Network flows: Notation

Constraints

Calibration

Balancing

Pictures of the network

Modeling

Networks economics I

Network economics II

- $X_{ae}^P$: catches (Sardara)
- $S_f^P$: commodity production, fresh or frozen (Fishstat)
- $I_f^P$: imports of commodity, fresh or frozen (Fishstat)
- $E_f^P$: exports of commodity, fresh or frozen (Fishstat)
- $S_c^P$: production of a prepared commodity from frozen fish (Fishstat)
- $I_c^P$: imports of a prepared commodity (Fishstat)
- $E_c^P$: exports of a prepared commodity (Fishstat)
The tuna commodity chain

Balancing data

Available data

Network flows: Notation

Constraints

Calibration

Balancing

Pictures of the network

Modeling

Networks economics I

Network economics II

\[ X_{ae}^p : \text{catches of species } e \text{ by country } p \text{ in area } a; \]

\[ X_{fe}^p : \text{from catches to fresh or frozen commodities}; \]

\[ X_{pp'}^{p_f} : \text{trade of fresh or frozen fish from } p \text{ to } p'; \]

\[ X_{fp}^p : \text{national consumption of fresh commodity } f \text{ by country } p; \]

\[ X_{fc}^p : \text{transformation of commodity } f \text{ in commodity } c \text{ by country } p; \]

\[ X_{pp'}^{p_c} : \text{trade of prepared fish from } p \text{ to } p'; \]

\[ X_{pc}^p : \text{national consumption of prepared commodity } c \text{ by country } p; \]
The tuna commodity chain

Balancing data

Available data

Network flows: Notation

Constraints

Constraints

Calibration

Constraints

Calibration

Balancing

Pictures of the network

Modeling

Networks economics I

Network economics II
Balancing data

Available data

Network flows: Notation

Constraints

Calibration

Balancing

Pictures of the network

Modeling

Networks economics I

Network economics II

The tuna commodity chain
The tuna commodity chain
Balancing data
Available data
Network flows: Notation
Constraints
Calibration
Constraints
Calibration
Balancing
Pictures of the network
Modeling
Networks economics I
Network economics II

Constraints

■ for all \( p \), \( X_{ef}^p > 0 \) only if \( K_{ef} = 1 \).
■ for all \( p \), \( X_{fc}^p > 0 \) only if \( H_{fc} = 1 \).
■ for all \( f \), \( X_{fp}^{pp'} > 0 \) only if \( L_{pp'}^F = 1 \).
■ for all \( f \), \( X_{cp}^{pp'} > 0 \) only if \( L_{pp'}^C = 1 \).
Calibration

The tuna commodity chain

Balancing data

Available data
Network flows: Notation

Constraints

Calibration

Balancing

Pictures of the network

Modeling

Networks economics I

Network economics II

- \( \overline{X_{ae}}^p \sim X_{ae}^p \) : catches (Sardara)
- \( \overline{S_f^p} \sim \sum_e X_{ef}^p \) : commodity production, fresh or frozen (Fishstat)
- \( \overline{I_f^p} \sim \sum_{p'} X_{f^p}^{p'} \) : imports of commodity, fresh or frozen (Fishstat)
- \( \overline{E_f^p} \sim \sum_{p'} X_{f^p}^{pp'} \) : exports of commodity, fresh or frozen (Fishstat)
- \( \overline{S_c^p} \sim \sum_f \theta_c X_{fc}^p \) : production of a prepared commodity from frozen fish (Fishstat)
- \( \overline{I_c^p} \sim \sum_{p'} X_{c^p}^{p'} \) : imports of a prepared commodity (Fishstat)
- \( \overline{E_c^p} \sim \sum_{p'} X_{c^p}^{pp'} \) : exports of a prepared commodity (Fishstat)
### The tuna commodity chain

#### Balancing data
- Available data

#### Network flows: Notation
- Constraints
- Constraints
- Calibration
- Calibration
- Balancing

#### Pictures of the network

#### Modeling
- Networks economics I
- Network economics II

#### Tables

<table>
<thead>
<tr>
<th></th>
<th>ALB,FRE</th>
<th>SKJ,FRE</th>
<th>TUN,FRE</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALB,FRE</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>ALB,FRO</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>BET,FRE</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>BET,FRO</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>BFT,FRE</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>BFT,FRO</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SKJ,FRE</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SKJ,FRO</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>SKJ,LOI</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>TUN,FRE</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>TUN,FRO</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>TUN,LOI</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>YFT,FRE</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>YFT,FRO</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
The tuna commodity chain

Balancing data

Available data

Network flows: Notation

Constraints

Calibration

Constraints

Calibration

Balancing

Pictures of the network

Modeling

Networks economics I

Network economics II

Calibration

- $\overline{X}_{ae}^{p}\simeq X_{ae}^{p}$: catches (Sardara)
- $\overline{S}_{f}^{p}\simeq \sum_{e} X_{ef}^{p}$: commodity production, fresh or frozen (Fishstat)
- $\overline{I}_{f}^{p}\simeq \sum_{p'} X_{fp'}^{p}$: imports of commodity, fresh or frozen (Fishstat)
- $\overline{E}_{f}^{p}\simeq \sum_{p'} X_{fp'}^{pp}$: exports of commodity, fresh or frozen (Fishstat)
- $\overline{S}_{c}^{p}\simeq \sum_{f} \theta_{c} X_{fc}^{p}$: production of a prepared commodity from frozen fish (Fishstat)
- $\overline{I}_{c}^{p}\simeq \sum_{p'} X_{cp'}^{p}$: imports of a prepared commodity (Fishstat)
- $\overline{E}_{c}^{p}\simeq \sum_{p'} X_{cp'}^{pp}$: exports of a prepared commodity (Fishstat)
Find \( X = (X_{ae}^p, X_{fe}^p, X_{ff}^{pp'}, X_{fc}^p, X_{cp}^{pp'}, X_{cc}^p) \) minimizing:

\[
S = \sum_{ae} (X_{ae}^p - X_{ae}^p)^2 + \sum_{fp} (S_{f}^p - \sum_{e} X_{ef}^p)^2 + \sum_{fp} (I_{f}^p - \sum_{p'} X_{fp}^{pp'})^2 + \sum_{fp} (E_{f}^p - \sum_{p'} X_{fp}^{pp'})^2 \\
+ \sum_{fp} (S_{c}^p - \theta_{c} \sum_{a} X_{fc}^p)^2 + \sum_{cp} (I_{c}^p - \sum_{p'} X_{cp}^{pp'})^2 + \sum_{cp} (E_{c}^p - \sum_{p'} X_{cp}^{pp'})^2
\]

under constraints:

- \( X_{ef}^p > 0 \) only if \( K_{ef} = 1 \), for all \( p, e, f \),
- \( X_{fc}^p > 0 \) only if \( H_{fc} = 1 \), for all \( p, f, c \),
- \( X_{ff}^{pp'} > 0 \) only if and \( L_{ff}^{pp'} = 1 \), for all \( f, p, p' \),
- \( X_{cc}^{pp'} > 0 \) only if and \( L_{cc}^{pp'} = 1 \), for all \( c, p, p' \),
- \( \sum_{e} X_{ef}^p + \sum_{p'} X_{fp}^{pp'} = \sum_{p'} X_{fp}^{pp'} + \sum_{c} \theta_{c} X_{fc}^p + X_{fp}^p \), for all \( p, f \),
- \( \sum_{f} X_{fc}^p + \sum_{p'} X_{cp}^{pp'} = \sum_{p'} X_{cp}^{pp'} + X_{cp}^p \), for all \( p, c \).
Pictures of the network
Trade fresh and frozen

NETWORK_ 1997

Fishing
Boats
Fish production
Fish consumption
Can production
Can consumption
Trade can

The tuna commodity chain
Balancing data
Pictures of the network
Catches
Trade fresh and frozen
Trade can
Spain
Spain
Japan
Japan
Modeling
Networks economics I
Network economics II

NETWORK_ 1997

Fishing
Boats
Fish production
Fish consumption
Can production
Can consumption
Spain

The tuna commodity chain
Balancing data
Pictures of the network
Catches
Trade fresh and frozen
Trade can
Spain
Spain
Japan
Japan
Modeling
Networks economics I
Network economics II

ESP 1997

- Fishing
- Boats
- Fish production
- Fish consumption
- Can production
- Can consumption
1. **Collapse of fresh market.** What could happen if, following a sanitary problem, there is a huge decrease of the demand for fresh tuna?

2. **Increase of oil price.** What could happen if, due to the decrease of supply, there is huge increase of oil prices, affecting both fishing costs and shipment costs?

3. **Trade regulation** What could happen if, in the continuity of these last years, there is a decrease of importation taxes for all tuna commodities?

4. **Global governance** What could happen if, a beginning of a global governance of tuna fisheries is set and results in the application of a system of fishing rights or fishing taxes?

5. **Climate change and productivity.** What could happen if, due to climate changes, productivity of marine areas changes dramatically, high or low?

6. **High seas marine protected areas (MPA).** What could happen if, a beginning of a global governance of tuna fisheries is set and results in the application of a system of marine protected areas?

7. **Moratory on fishing aggregative devices (FAD).** What could happen if, in the framework towards a global governance, it is decided to prohibit FAD for some years, affecting catchability.
The tuna commodity chain
Balancing data
Pictures of the network
Modeling
Scenarios
Nodes
Links
Modeling
Constraints on flows
Network economics: Equilibrium
Dynamics
Model calibration
Model calibration
Setting scenarios
Networks economics I
Networks economics II
### Nodes

<table>
<thead>
<tr>
<th>Node</th>
<th>Kind</th>
<th>Notation</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fish populations</td>
<td>Producer</td>
<td>$N_{ae}$</td>
<td>Stock $S_{ae}$</td>
</tr>
<tr>
<td>National fleets</td>
<td>Wholesaler</td>
<td>$N^p_e$</td>
<td>Fishing capacity $V^p_e$, depreciation rate $\eta^p_e$, investment rate $\sigma^p_e$</td>
</tr>
<tr>
<td>National fresh or frozen commodities</td>
<td>Wholesaler</td>
<td>$N^p_f$</td>
<td></td>
</tr>
<tr>
<td>production systems</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>National canned commodities</td>
<td>Wholesaler</td>
<td>$N^p_c$</td>
<td>Production capacity $U^p_c$, depreciation rate $\eta^p_c$, investment rate $\sigma^p_c$</td>
</tr>
<tr>
<td>production systems</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>National fresh or frozen commodities</td>
<td>Retailer</td>
<td>$M^p_f$</td>
<td>Inverse demand function $P^p_f = a^p_f - \frac{b^p_f L^p_f}{L^p_f}$</td>
</tr>
<tr>
<td>markets</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>National canned commodities</td>
<td>Retailer</td>
<td>$M^p_c$</td>
<td>Inverse demand function $P^p_c = a^p_c - \frac{b^p_c L^p_c}{L^p_c}$</td>
</tr>
<tr>
<td>markets</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Links

**The tuna commodity chain**

- Balancing data
- Pictures of the network

**Modeling**

- Scenarios
- Nodes
- Links

**Network economics I**

- Constraints on flows
- Network economics: Equilibrium
- Dynamics
- Model calibration

**Network economics II**

- Setting scenarios

---

<table>
<thead>
<tr>
<th>Link</th>
<th>Nodes</th>
<th>Notation</th>
<th>Characteristics, costs $C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Catches</td>
<td>$N_{ae} \rightarrow N_{p}^{e}$</td>
<td>$X_{ae}^{p}$</td>
<td>$C_{ae}^{p} = CF_{ae}^{p} + CK_{ae}^{p}/S_{ae}$, catchability $q_{ae}^{p}$</td>
</tr>
<tr>
<td>Destination of catches to fresh or frozen commodities</td>
<td>$N_{e}^{p} \rightarrow N_{f}^{p}$</td>
<td>$X_{ef}^{p}$</td>
<td>$C_{ef}^{p} = 0$</td>
</tr>
<tr>
<td>Production of canned commodities</td>
<td>$N_{f}^{p} \rightarrow N_{c}^{p}$</td>
<td>$X_{fc}^{p}$</td>
<td>$C_{fc}^{p}$ (including cannery costs)</td>
</tr>
<tr>
<td>Trade of fresh, frozen commodities between countries</td>
<td>$N_{f}^{p} \rightarrow N_{f}^{q}$</td>
<td>$X_{f}^{pq}$</td>
<td>$C_{f}^{pq}$</td>
</tr>
<tr>
<td>Trade of canned commodities between countries</td>
<td>$N_{c}^{p} \rightarrow N_{c}^{q}$</td>
<td>$X_{c}^{pq}$</td>
<td>$C_{f}^{pq}$</td>
</tr>
<tr>
<td>Local consumption fresh or frozen commodities</td>
<td>$N_{f}^{p} \rightarrow M_{f}^{p}$</td>
<td>$L_{f}^{p}$</td>
<td>$C_{f}^{p}$</td>
</tr>
<tr>
<td>Local consumption of canned commodities</td>
<td>$N_{c}^{p} \rightarrow M_{c}^{p}$</td>
<td>$L_{c}^{p}$</td>
<td>$C_{c}^{p}$</td>
</tr>
</tbody>
</table>
At time $t$,

1. set parameter values according to scenario.

2. compute the economic equilibrium of the system:
   
   $$(\text{Fishing capacity, Stocks, Demand, Costs, Parameters}) \rightarrow (\text{Catches, Sales, Prices}).$$

From time $t$ to time $t + 1$, compute the new state of the system

1. $$(\text{Stock, Catches, Parameters}) \rightarrow (\text{Stock}).$$

2. $$(\text{Sales, Trade, Prices, Costs, Parameters}) \rightarrow (\text{Income}).$$

3. $$(\text{Income, Fishing capacity, Canning capacity, Parameters}) \rightarrow (\text{Fishing capacity, Canning capacity}).$$
■ Constraints due to limited fishing capacity:

- Catches $X_{ae}^p$ are related to stocks $S_{ae}$ and efforts $V_{ae}^p$
  \[ X_{ae}^p = q_{ae}^p S_{ae} V_{ae}^p. \]

- Efforts $V_{ae}^p$ of a fleet are limited by fishing capacity
  \[ V_e^p : \sum_a V_{ae}^p \leq V_e^p. \]

- Thus \[ \sum_a X_{ae}^p / (q_{ae}^p S_{ae}) \leq V_e^p. \]

■ Constraints on can production capacity: \[ \sum_f X_{fc}^p \leq U_c^p. \]
A Walras’ equilibrium on markets (either inflow is greater than outflow and there is no price or inflow equals outflow and there is a price),

Walras’ equilibrium on links (either price at origin plus transportation costs are greater than price at destination and there is no flow, or price at origin plus transportation costs equals price at destination and there is flow smaller than capacity, or price at origin plus transportation costs are greater than price at destination and flow equals capacity).
Dynamics

The tuna commodity chain
Balancing data
Pictures of the network
Modeling
Scenarios
Nodes
Links
Modeling Constraints on flows
Network economics: Equilibrium
Dynamics
Model calibration
Model calibration
Setting scenarios
Networks economics I
Network economics II

- **Stocks**: \( S_{ae} \rightarrow S_{ae} + r_{ae}(1 - S_{ae}/K_{ae}) - \sum_p X_{ae}^p \)

- **Fisheries income**: \( I_{pe}^p = \sum_f X_{ef}^p P_f^p - \sum_a X_{ae}^p P_{ae}^p \)

- **Fishing capacity**: \( V_{pe}^p \rightarrow V_{pe}^p - \eta_{pe}^p V_{pe}^p + \sigma_{pe}^p I_{pe}^p \)

- **Cannery income**: \( I_{pc}^p = \sum_q X_{cq}^p P_f^p - \sum_q X_{cqp}(P_f^q + C_{cqp}) - \sum_{fc} X_{fc}^p P_{ae}^p \)

- **Cannery capacity**: \( U_{pc}^p \rightarrow U_{pc}^p - \eta_{pc}^p U_{pc}^p + \sigma_{pc}^p I_{pc}^p \)
### Model calibration

#### The tuna commodity chain

Balancing data

#### Pictures of the network

Modeling

#### Scenarios

Nodes

Links

Modeling

Constraints on flows

Network economics: Equilibrium

Dynamics

#### Model calibration

Setting scenarios

#### Model calibration

Modeling

#### Settings

Model calibration

##### Setting scenarios

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Meaning and estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial stock</td>
<td>$S_{ae}$</td>
<td>From Sardara</td>
</tr>
<tr>
<td>Carrying capacity</td>
<td>$K_{ae}$</td>
<td>From Sardara</td>
</tr>
<tr>
<td>Renewal rate</td>
<td>$r_{ae}$</td>
<td>From Sardara</td>
</tr>
<tr>
<td>Fishing capacity</td>
<td>$V^P_e$</td>
<td>From Sardara</td>
</tr>
<tr>
<td>Fishing capacity</td>
<td>$V^P_e$</td>
<td>From Sardara</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>$\eta^P_e$</td>
<td></td>
</tr>
<tr>
<td>Investment rate</td>
<td>$\eta^P_e$</td>
<td></td>
</tr>
<tr>
<td>Demand function parameters</td>
<td>$a_n, b_n$</td>
<td>from observed prices</td>
</tr>
</tbody>
</table>

Parameter | Notation | Meaning and estimation |
### Parameter | Notation | Meaning and estimation
--- | --- | ---
Fishing costs | $C_{ae}^p$ | $C_{ae}^p = C F_{ae}^p + C K_{ae}^p / S_{ae}$; a part related to catches, a part related to effort (decreasing with fish abundance)
Canning costs | $C_{fc}^p$ | from surveys (and from differences between observed prices)
Trading costs | $C_{f}^{pq}$ | from surveys (and from differences between observed prices)
Trading costs | $C_{c}^{pq}$ | from surveys (and from differences between observed prices)
### Setting scenarios

#### The tuna commodity chain

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Petrol price</td>
<td>$\pi$</td>
<td>$C_l \rightarrow \pi C_l$ for $l = (aep)$ (fishing costs)</td>
</tr>
<tr>
<td>Marine protected areas</td>
<td>$\theta$</td>
<td>$C_l \rightarrow \theta C_l$ for $l = (aep)$ (fishing costs in several areas of Indian Oceans)</td>
</tr>
<tr>
<td>Productivity changes</td>
<td>$\lambda$</td>
<td>$K_{ae} \rightarrow \lambda K_{ae}$ with $K_{ae}$ carrying capacity</td>
</tr>
<tr>
<td>Globalization</td>
<td>$\gamma$</td>
<td>$C_l \rightarrow \gamma C_l$ for $l = f pq$, or $l = c pq$ (trading costs)</td>
</tr>
<tr>
<td>Demand changes for fresh or frozen fish</td>
<td>$\delta$</td>
<td>$a_f^p \rightarrow \delta a_f^p$</td>
</tr>
<tr>
<td>Capturability changes</td>
<td>$\sigma$</td>
<td>$q_{ae}^p \rightarrow \sigma q_{ae}^p$</td>
</tr>
</tbody>
</table>
Networks economics I

Anna Nagurney

Network structure
Nodes and links
Characteristics
Equilibrium
Variational inequality

Networks economics II
Network structure

The tuna commodity chain
Balancing data
Pictures of the network
Modeling
Networks economics I
Anna Nagurney
Network structure
Nodes and links
Characteristics
Equilibrium
Variational inequality
Networks economics II
Nodes and links

- **Nodes** $n \in N$ represent agents:
  - producers ($p \in P$),
  - wholesalers ($g \in G$),
  - retailers ($d \in D$).

- **Links** $l \in L$ represent flows between agents.

- **Incidence**
  - $o(l)$ the origin of a node $l$.
  - $d(l)$ the destination of a node $l$.
  - Incidence functions: $\delta_n^o = 1$ if $o(l) = n$, $\delta_n^d = 1$ if $d(l) = n$.
Characteristics of a link $l \in L$ are:

- commodity flow: $X_l$,
- transportation costs $C_l$
- capacity $M_l$ (maximum possible flow).

Characteristics of a node $n \in N$ are:

- ingoing flow $I_n = \sum_l \delta^d_{nl} X_l$,
- outgoing flow $O_n = \sum_l \delta^o_{nl} X_l$
- commodity price $P_n$.

- For $n \in D$, we have $P_n = F(I_n)$; we use $P_n = a_n - b_n I_n$; parameters $a_n$ and $b_n$ are given.
- For $n \in G$, $P_n$ results from market equilibrium.
- For $n \in P$, $P_n$ equals production costs (given).
The state of the system is defined as:

$Z = ((X_l), (P_n))$.

To a state $Z$, we associate:

- $E_n(Z) = \sum_l (\delta^d_{nl} - \delta^o_{nl}) X_l$
- $A_l(Z) = P_{o(l)} - P_{d(l)} - C_l = \sum_n (\delta^o_{nl} - \delta^d_{nl}) P_n - C_l$

Network equilibrium is a state of the system $Z^* = ((X^*_l), (P^*_n))$ such that:

- For all wholesalers $n \in G$,
  - either $P^*_n = 0$ and $E_n(Z^*) \geq 0$;
  - or $P^*_n > 0$ and $E_n(Z^*) = 0$;

- For all links $l$,
  - either $X^*_l = 0$ and $A_l(Z^*) \geq 0$;
  - or $0 < X^*_l < M_l$ and $A_l(Z^*) = 0$;
  - or $X^*_l = M_l$ and $A_l(Z^*) \leq 0$;
Variational inequality

- Constrained set: $K = \prod_l [0, M_l] \times R^N_G \subset R^{N_L} \times R^{N_G}$

- Functional $F : K \rightarrow R^{N_L} \times R^{N_G}$, $Z = ((X_l), (P_n)) \rightarrow F(Z) = ((A_l(Z), E_n(Z))$. 

- $Z^* \in K$ is an equilibrium state of the system if and only if it is a solution of the variational inequality $VI(F, K)$: find $Z^* \in K$ such that for all $Z \in K$, $(Z - Z^*).F(Z^*) \geq 0$

- $F$ is an affine function: $F(Z) = M.Z + N$

$$M = \begin{pmatrix} M_{LL} & M_{LN} \\ -T & 0 \end{pmatrix}, \quad N = \begin{pmatrix} N_L \\ 0 \end{pmatrix}$$

- $N_l = -C_l - \sum_{n \in P} \delta^o_{nl} C_n + \sum_{n \in D} \delta^o_{nl} a_n$

- $M_{ll'} = \sum_{n \in D} (\delta^d_{nl'} \delta^d_{nl}) b_n$

- $M_{ln} = \delta^o_{nl} - \delta^d_{nl}$

- $M$ is a non negative matrix.
Network economics II
Characteristics of a link \( l \in L \) are:

- commodity flow: \( X_l \),
- transportation costs \( C_l \)

Characteristics of a node \( n \in N \) are:

- ingoing flow \( I_n = \sum_l \delta^d_{nl} X_l \),
- outgoing flow \( O_n = \sum_l \delta^o_{nl} X_l \)
- commodity price \( P_n \).

For \( n \in D \), we have \( P_n = F(I_n) \); we use \( P_n = a_n - b_n I_n \); parameters \( a_n \) and \( b_n \) are given.

For \( n \in G \), \( P_n \) results from market equilibrium.

For \( n \in P \), \( P_n \) equals production costs (given).

- production constraints: \( \sum_l \delta^d_{ql} f_{ql} X_l \leq g_q \) for \( q \in Q \subset G \subset N \), \( f_{ql} \geq 0 \).
A state of the system is $Z = ((X_l), (P_n), (\lambda_q)) \in H = \mathbb{R}^{NL}_+ \times \mathbb{R}^{NG}_+ \times \mathbb{R}^{NQ}_+$; $\lambda_q$ is a shadow price associated to constraint $q$.

We denote: $D_q(Z) = g_q - \sum_l \delta_{ql} f_{ql} X_l$.

A network equilibrium is a state $Z^* = ((X_l^*), (P_n^*), (\lambda_q^*)) \in H$ such that:

- For all wholesalers $n \in G$, 
  - either $P_n^* = 0$ and $E_n(Z^*) \geq 0$;
  - or: $P_n^* > 0$ and $E_n(Z^*) = 0$;

- For all links $l$ such that $d(l) \notin Q$,
  - either $X_l^* = 0$ and $A_l(Z^*) \geq 0$;
  - or $0 < X_l^*$ and $A_l(Z^*) = 0$;

- For all $l \in L$ such that $q = d(l) \in Q$,
  - either $X_l^* = 0$ and $A_l(Z^*) \geq \lambda_q^*$;
  - or $0 < X_l^*$ and $A_l(Z^*) = \lambda_q^*$;
  - moreover $\lambda_q^* = 0$ if $D_q(Z^*) > 0$. 


The constrained set is: $H = R^N_{+} \times R^G_{+} \times R^Q_{+}$

We put: $B_l(Z) = A_l(Z) + F_l(Z)$, $F_l(z) = \sum_q \delta^q_l f_q \lambda_q$.

Functional $F : H \rightarrow R^L_{+} \times R^G_{+} \times R^Q_{+}$ is defined by $F(Z) = ((B_l(Z), E_n(Z), D_q(Z))$.

$Z^* \in H$ is an equilibrium state of the system if and only if it is a solution of the variational inequality $VI(F,H)$: find $Z^* \in H$ such that for all $Z \in H$, $(Z - Z^*).F(Z^*) \geq 0$

$F$ is an affine function: $F(Z) = M.Z + N$ with

\[
M = \begin{pmatrix}
M_{LL} & M_{LN} & M_{LQ} \\
-TM_{LN} & 0 & 0 \\
-TM_{LQ} & 0 & 0
\end{pmatrix}, \quad N = \begin{pmatrix}
N_L \\
0 \\
N_Q
\end{pmatrix}
\]

$N_l = -C_l - \sum_{n \in P} \delta^o_{nl} C_n + \sum_{n \in D} \delta^o_{nl} a_n, N_q = g_q,$

$M_{ll'} = \sum_{n \in D} (\delta^{d}_{nl'} \delta^{d}_{nl}) b_n; M_{ln} = \delta^o_{nl} - \delta^d_{nl}; M_{lq} = \delta^d_{ql} f_{ql}$.

$M$ is a non negative matrix.
The tuna commodity chain

The tuna commodity chain

Nodes: \( N = \{ N_{ae}, N_{ep}, N_{fp}, N_{cp}, M_{fp}, M_{cp} \} \).

- Producers: \( P = \{ N_{ae} \} \).
- Wholesalers \( G = \{ N_{ep}, N_{fp}, N_{cp} \} \).
- Constrained wholesalers \( Q = \{ N_{ep}, N_{cp} \subset G \} \).
- Retailers: \( D = \{ M_{fp}, M_{cp} \} \).

Links:

\[
L = L_{AEP} \cup L_{EFP} \cup L_{FCP} \cup L_{FPQ} \cup L_{CPQ} \cup L_{FP} \cup L_{CP}
\]
M = \begin{pmatrix} M_{LL} & M_{LN} & M_{LQ} \\ -T M_{LN} & 0 & 0 \\ -T M_{LQ} & 0 & 0 \end{pmatrix}, \quad N = \begin{pmatrix} N_L \\ 0 \\ N_Q \end{pmatrix}

N_L: N_{aep} = -P_{ae} - C_{aep}, N_l = -C_l \text{ for } l \notin L_{AEP}.

N_Q: N_{ep} = V^p, N_{cp} = U^p.

M_{LL}: M_{ll'} = 0 \text{ except } M_{fp}(fp) = b_{fp}, M_{cp}(cp) = b_{cp},

M_{LN}: M_{ln} = \delta^o_{nl} - \delta^d_{nl}; \text{ thus } M_{ln} = 0 \text{ except }

\begin{align*}
\boxminus M_{aep}(ep) &= M_{ep}(fp) = M_{cp}(cp) = M_{pq}(pq) = M_{cpq}(cq) = -1, \\
\boxminus M_{efp}(ef) &= M_{fp}(fc) = M_{fpq}(fp) = M_{cpq}(cp) = 1 \\
\boxminus M_{fp}(fp) &= M_{cp}(cp) = 1
\end{align*}

M_{LQ}: M_{ql} = \delta^d_{ql} f_{ql}, \text{ thus } M_{ql} = 0 \text{ except }

\begin{align*}
\boxminus M_{aep}(ep) &= 1/(q^p_{ae} S_{ae}) \\
\boxminus M_{fcp}(cp) &= 1
\end{align*}